A new kind of weak-coupling in top-quark physics?

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In the standard model (SM), for the $t \to W + b$ decay mode, the relative phase is 0^o between the dominant $A(0, -1/2)$ and $A(-1, -1/2)$ helicity amplitudes. However, in the case of an additional large $t_R \to b_L$ chiral weak-transition moment, there is instead a 180^o relative phase and three theoretical numerical puzzles. This phase can be measured at the Tevatron or LHC in top-antitop pair production by use of W-boson longitudinal-transverse interference in beam-referenced stage-two spin-correlation functions. Indeed, this is a new type of weak-coupling for it is directly associated with E_W, the W-boson energy in the top quark rest frame, instead of with a canonical effective mass scale. For most $2 \to 2$ reactions, the simple off-shell continuation of this additional coupling is found to have good high energy properties, i.e. it does not destroy 1-loop unitarity of the SM. In a subset of processes, additional third-generation couplings are required.

1. Does the top-quark have a large chiral weak-transition moment in $t \to W^+ b$ decay?

By W-boson longitudinal-transverse quantum interference, the relatively simple 4-angle beam-referenced stage-two spin-correlation function

$$\mathcal{G}|_0 + \mathcal{G}|_{sig} \tag{1}$$

enables measurement of the relative phase of the 2 dominant amplitudes in $t \to W^+ b$ decay with both gluon and quark production contributions [1-4].

In the standard model, for the $t \to W^+ b$ decay mode, the relative phase is 0^o between the dominant $A(0, -1/2)$ and $A(-1, -1/2)$ helicity amplitudes for the standard model $V - A$ coupling.

For the case of an additional chiral-tensorial-coupling in $g_L = g_{f_M + f_E} = 1$ units,

$$\frac{1}{2}\Gamma^\mu = g_L \left[\gamma^\mu P_L + \frac{1}{2\Lambda_+}\imath\sigma^{\mu\nu}(q_W)_\nu P_R\right] = P_R\left[\gamma^\mu + \imath\sigma^{\mu\nu}\upsilon_\nu\right] \tag{2}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and $\Lambda_+ = E_W/2 \sim 53GeV$ in the top rest frame. In the case of such an additional large $t_R \to b_L$ chiral weak-transition moment, there is instead a 180^o relative phase between the $A(0, -1/2)$ and $A(-1, -1/2)$ helicity amplitudes. The associated on-shell partial-decay-width $\Gamma(t \to W^+ b)$ does differ for these two Lorentz-invariant couplings [$\Gamma_{SM} = 1.55GeV$, $\Gamma_+ = 0.66GeV$ versus less than $12.7GeV@95\%C.L.$ in CDF conf. note 8953(2007) in PDG2008].

2. Helicity decay amplitudes

In the t_1 rest frame, the matrix element for $t_1 \to {W_1}^+ b$ is

$$\langle\theta_1^t, \phi_1, \lambda_{W^+}, \lambda_b|\frac{1}{2}, \lambda_1\rangle = D^{(1/2)*}_{\lambda_1,\mu}(\phi_1, \theta_1^t, 0)A\left(\lambda_{W^+}, \lambda_b\right) \tag{3}$$

where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the ${W_1}^+$ and b-quark helicities. An asterisk denotes complex conjugation. The final ${W_1}^+$ momentum is in the θ_1^t, ϕ_1 direction and the b-quark momentum is in the opposite direction. We use the Jacob-Wick phase convention.

So there are 4 moduli and 3 relative phases to be measured, see refs. in [1]. Both in the SM and in the case of an additional large $t_R \to b_L$ chiral weak-transition moment, the $\lambda_b = -1/2$ and $\lambda_{\bar{b}} = 1/2$ amplitudes are more than ~ 30 times larger than the $\lambda_b = 1/2$ and $\lambda_{\bar{b}} = -1/2$ amplitudes.

	$A(0,-\frac{1}{2})$	$A(-1,-\frac{1}{2})$	$A(0,\frac{1}{2})$	$A(1,\frac{1}{2})$
A in $g_L = 1$ units:				
$(V - A)$	338	220	-2.33	-7.16
$(V - A) + (t_R \dashrightarrow b_L)$	220	-143	1.52	-4.67
A_{New}				
$(V - A)$	0.84	0.54	-0.0058	-0.018
$(V - A) + (t_R \dashrightarrow b_L)$	0.84	-0.54	0.0058	-0.018

Table I: Helicity amplitudes for $(V-A)$ coupling and the $(+)$ coupling of Eq.(2). Note $A_{New} = A_{g_L=1}/\sqrt{\Gamma}$.

3. Idea of a W-boson Longitudinal-Transverse interference measurement

For the charged-lepton-plus-jets reaction pp or $p\bar{p} \to t\bar{t} \to (W^+b)(W^-\bar{b}) \to (l^+\nu b)(W^-\bar{b})$, one selects a "signal contribution" so that its intensity-observable is the product of an amplitude in which the W^+ is longitudinally-polarized with the complex-conjugate of an amplitude in which the W^+ is transversely polarized, summed with the complex-conjugate of this product. The helicity formalism is a general method for investigating applications of W-boson interference in stage-two spin-correlation functions for describing the charged-lepton plus jets channel, and for the di-lepton plus jets channel.

4. Observables and signatures

The 2 dominant polarized partial widths are

$$\Gamma(0,0) \equiv |A(0,-1/2)|^2 , \; \Gamma(-1,-1) \equiv |A(-1,-1/2)|^2 \tag{4}$$

The 2 W-boson Longitudinal-Transverse interference widths are

$$\begin{aligned} \Gamma_R(0,-1) &= \Gamma_R(-1,0) \equiv Re[A(0,-1/2)A(-1,-1/2)^*] \\ &\equiv |A(0,-1/2)||A(-1,-1/2)| \cos\beta_L \qquad (5) \\ \Gamma_I(0,-1) &= -\Gamma_I(-1,0) \equiv Im[A(0,-1/2)A(-1,-1/2)^*] \\ &\equiv -|A(0,-1/2)||A(-1,-1/2)| \sin\beta_L \qquad (6) \end{aligned}$$

The relative phase of these 2 dominant amplitudes is β_L. In both models

$$Probability\ W_L = \frac{\Gamma(0,0)}{\Gamma} = 0.70 \tag{7}$$

$$Probability\ W_T = \frac{\Gamma(-1,-1)}{\Gamma} = 0.30 \tag{8}$$

But there are the respective signatures

$$\eta_L \equiv \frac{\Gamma_R(0,-1)}{\Gamma} = +0.46\,(SM)$$

for the Standard Model, and

$$\eta_L = -\,0.46\,(+)$$

for the case of an additional large chiral weak-transition moment. In both models, unless there is a violation of time-reversal invariance

$$\eta_L^{'} \equiv \frac{\Gamma_I(0,-1)}{\Gamma} = 0 \qquad (9)$$

5. Definition of angles in spin-correlation function

For pp or $p\bar{p} \rightarrow t\bar{t} \rightarrow (W^+b)(W^-\bar{b}) \rightarrow (l^+\nu b)(W^-\bar{b})$, the "Spin-Correlation Function" depends on four angles. While the cosine of the **4th angle**, $\cos\Theta_B$, can be integrated out, the expressions are clearer if it is not. Θ_B is the "beam referencing angle" in the $(t\,\bar{t})_{cm}$ frame [3,4].

The 3 angles are:

(i) The spherical angles θ_a and ϕ_a which specify the final positive-charged lepton in the W_1^+ rest frame when the boosts are from the $(t\,\bar{t})_{cm}$ frame to the t_1 frame and then to the W_1^+ rest frame. (ii) The cosine of the polar

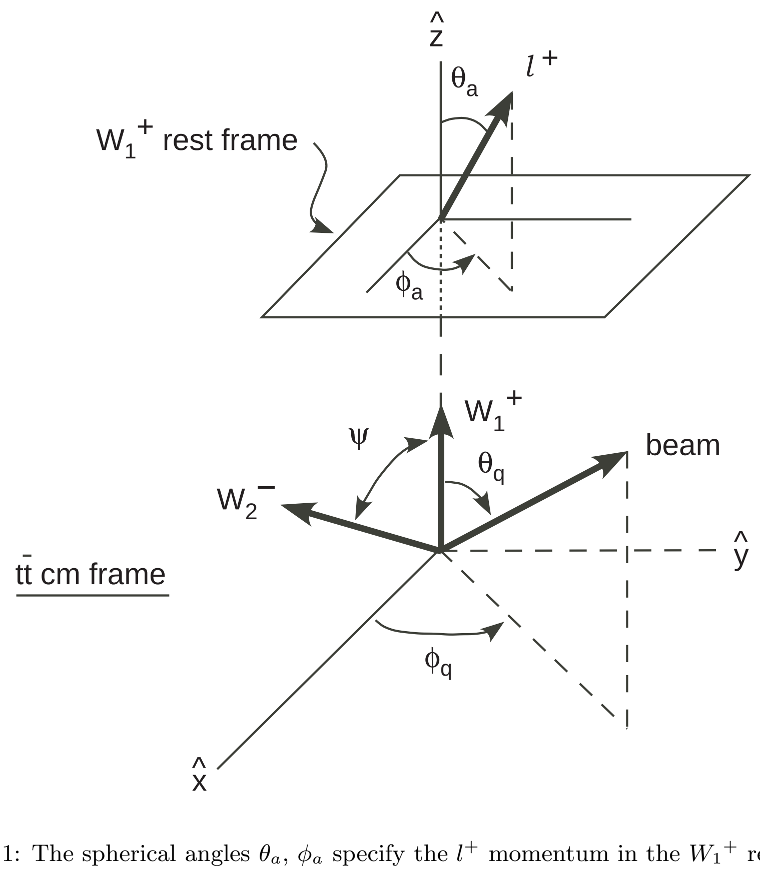

Figure 1: The spherical angles θ_a, ϕ_a specify the l^+ momentum in the $W_1{}^+$ rest frame.

angle θ_2^t to specify the W_2^- momentum direction in the anti-top rest frame. Usage of $\cos\theta_2^t$ is equivalent to using the $(t\,\bar{t})_{cm}$ energy of this hadronically decaying W_2^- .

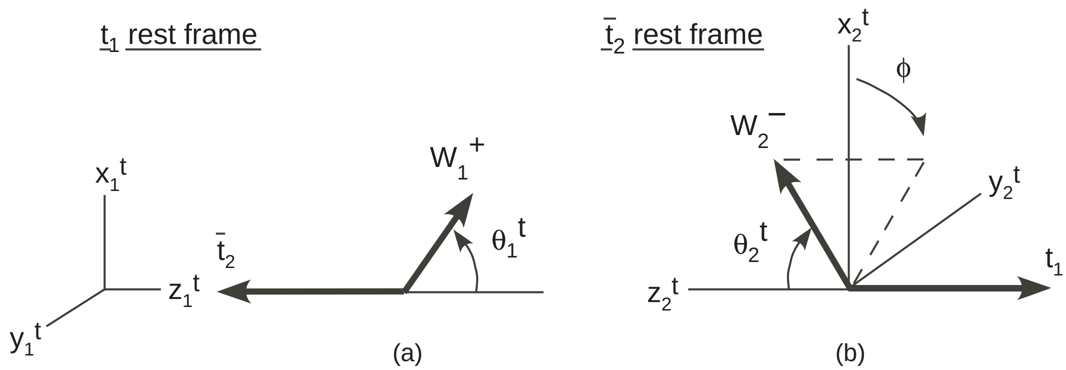

Figure 2: Summary illustration showing the three angles θ_1^t, θ_2^t and ϕ describing the first stage in the sequential-decays of the $t\bar{t}$ system in which $t_1 \rightarrow {W_1}^+ b$ and $\bar{t_2} \rightarrow {W_2}^- \bar{b}$. In (a) the b momentum, not shown, is back to back with the ${W_1}^+$. In (b) the $\bar{b}$ momentum, not shown, is back to back with the ${W_2}^-$.

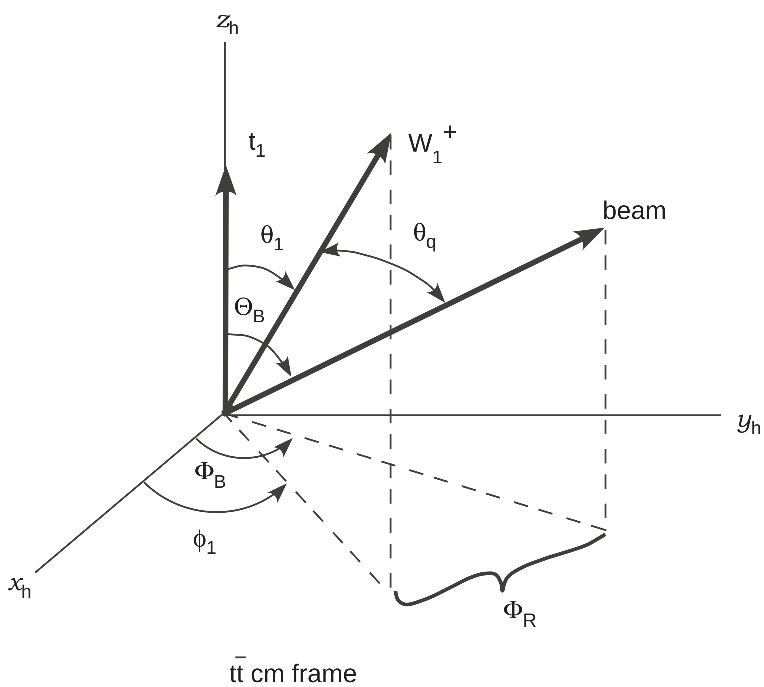

Figure 3: The g_1 gluon-momentum or q_1 quark-momentum "beam" direction is specified by the spherical angles Θ_B, Φ_B. $\cos\Theta_B$ can be integrated out.

6. Spin-correlation function

By W-boson longitudinal-transverse interference, the relatively simple 4-angle beam-referenced stage-two spin-correlation function

$$\mathcal{G}|_0 + \mathcal{G}|_{sig} \tag{10}$$

enables measurement of the relative phase of the 2 dominant amplitudes in $t \rightarrow W^+ b$ decay:

The "background term" is

$$\mathcal{G}|_0 \;=\; B(s,\Theta_B)|_0 \left\{ \frac{1}{2}\Gamma(0,0)\sin^2\theta_a + \Gamma(-1,-1)\sin^4\frac{\theta_a}{2} \right\} \tag{11}$$

The “signal term” is

$$\mathcal{G}|_{sig} = -\frac{\pi}{2\sqrt{2}} B(s,\Theta_B)|_{sig} \cos\theta_2^t \sin\theta_a \sin^2\frac{\theta_a}{2} \{\Gamma_R(0,-1)\cos\phi_a - \Gamma_I(0,-1)\sin\phi_a\}\,\mathcal{R} \tag{12}$$

The signal contribution is suppressed by by the factor $\mathcal{R} = (\texttt{prob}\,W_L) - (\texttt{prob}\,W_T) = 0.40$. Eqs (11,12) omit a common overall factor $\frac{16\pi^3 g^4}{9s^2}[\overline{\Gamma}(0,0)+\overline{\Gamma}(1,1)]$, see [3,4].

Beam-Referencing Factors:

In Eq.(10), one adds the quark and the gluon production contributions with for quark-production

$$B_0^q(s,\Theta_B) = \frac{1}{24}[1+\cos^2\Theta_B + \frac{4m^2}{s}\sin^2\Theta_B]$$
$$B_{sig}^q(s,\Theta_B) = \frac{1}{24}[1+\cos^2\Theta_B - \frac{4m^2}{s}\sin^2\Theta_B] \tag{13}$$

and for gluon-production

$$B_0^g(s,\Theta_B) = \overline{c}(s,\Theta_B)[\sin^2\Theta_B(1+\cos^2\Theta_B) + \frac{8m^2}{s}(\cos^2\Theta_B + \sin^4\Theta_B) - \frac{16m^4}{s^2}(1+\sin^4\Theta_B)]$$
$$B_{sig}^g(s,\Theta_B) = \overline{c}(s,\Theta_B)[\sin^2\Theta_B(1+\cos^2\Theta_B) - \frac{8m^2}{s}(1+\sin^2\Theta_B) + \frac{16m^4}{s^2}(1+\sin^4\Theta_B)] \tag{14}$$

where the overall gluon-pole-factor

$$\overline{c}(s,\Theta_B) = \frac{3s^2g^4}{96(m^2-t)^2(m^2-u)^2}\,[7+\frac{36p^2}{s}\cos^2\Theta_B] \tag{15}$$

depends on the $(t\bar{t})_{c.m.}$ center-of-momentum energy $\sqrt{s}$ and $\cos\Theta_B$, and includes the gluon color factor. In application, for instance to $pp \to t\bar{t}X$, parton-level top-quark spin-correlation functions need to be smeared with the appropriate parton-distribution functions with integrations over $\cos\Theta_B$ and the $(t\bar{t})_{c.m.}$ energy, $\sqrt{s}$.

There is a common final-state interference structure in these BR-S2SC functions for the charged-lepton plus jets reaction pp or $p\bar{p} \to t\bar{t} \to \ldots$. From (11,12), the final-state relative phase effects do not depend on whether the final $t_1\bar{t}_2$ system has been produced by gluon or by quark production.

Measurement of the sign of the $\eta_L \equiv \frac{\Gamma_R(0,-1)}{\Gamma} = \pm 0.46(\mathrm{SM}/+)$ helicity parameter could exclude a large chiral weak-transition moment in favor of the SM prediction.

7. The three theoretical numerical puzzles

These puzzles arose in a general search for empirical ambiguities between the SM's $(V-A)$ coupling and possible single additional Lorentz couplings that could occur in top-quark decay experiments, see refs. in [1].

1st Puzzle's associated phenomenological m_{top} mass formula:

With ICHEP2008 empirical mass values $(m_t = 172.4 \pm 1.2GeV, m_W = 80.413 \pm 0.048GeV)$

$$y = \frac{m_W}{m_t} = 0.4664 \pm 0.0035 \tag{16}$$

This can be compared with the amplitude equality in the upper part of “Table 1”, see the corresponding two “220” entries,

$$A_+(0,-1/2) = a\,A_{SM}(-1,-1/2) \tag{17}$$

with $a = 1 + O(v \neq y\sqrt{2}, x)$. By expanding in the mass ratio $x^2 = (m_b/m_t)^2 = 7 \cdot 10^{-4}$,

$$1 - \sqrt{2}y - y^2 - \sqrt{2}y^3 = x^2(\frac{2}{1-y^2} - \sqrt{2}y) - x^4(\frac{1-3y^2}{(1-y^2)^3}) + \dots$$
$$= 1.89x^2 - 0.748x^4 + \dots$$

The only real-valued solution to this cubic equation is $y = 0.46006$ ($m_b = 0$).

Resolution of 2nd and 3rd Puzzles:

In the lower part of "Table 1", the two R-handed helicity b-quark amplitudes $A_{New} = A_{g_L=1}/\sqrt{\Gamma}$ have the same magnitude in the SM and the (+) model.

As a consequence of Lorentz-invariance, for the $t \rightarrow W^+ b$, the 4 intensity-ratios, $\Gamma_{L,T}|_{\lambda_b=\mp\frac{1}{2}}/\Gamma(t \rightarrow W^+ b)$ are identical for the standard model $V-A$ coupling and for the case of an additional chiral weak-moment of relative strength $\Lambda_+ = E_W/2$. This intensity-ratio equivalence does not depend on the numerical values of $y = \frac{m_W}{m_t}$ or $x = m_b/m_t$.

8. High energy properties of off-shell continuation

A simple off-shell continuation of Eq(2) is the $t \rightarrow W^+ b$ vertex

$$\frac{1}{2}\Gamma^\mu = g_L \left[\gamma^\mu P_L + \frac{m_t}{p_t \cdot q_W} \imath \sigma^{\mu\nu} (q_W)_\nu P_R \right] \tag{18}$$

To lowest order, this additional coupling has good high energy properties, i.e. it does not destroy 1-loop unitarity, for the processes $t\bar{t} \rightarrow W^+ W^-$ and $b\,\bar{b} \rightarrow W^- W^+$ with both W-bosons with longitudinal polarization, nor with either or both W-bosons with transverse polarizations. There are of course no effects to lowest order for their analogues with a $Z^o Z^o$ final state, nor for the processes like $t\,\bar{b} \rightarrow e^+ \nu_e$. By power-counting, versus the SM's fermion cancellations, there are no new effects for the ABJ gauge anomalies.

The processes $t\,\bar{b} \rightarrow W^+ \gamma, t\,\bar{b} \rightarrow W^+ H$ with H a Higgs boson, are not divergent. However, the additional gauge-vertex diagram for the process $t\,\bar{b} \rightarrow W^+ Z^o$ with longitudinal gauge-boson polarizations is linearly divergent in the large E_t limit in the $(t\,\bar{b})_{cm}$ frame. Since this divergence involves a denominator factor of $p_t \cdot (k_W + k_Z)$, instead of additional neutral current couplings to the third-generation fermions, we introduce an additional more-massive $X^\pm$ boson to cancel this divergence. The additional tXb vertex is

$$\frac{1}{2}\Gamma^\mu_X = -g_L \left[\frac{m_t}{p_t \cdot q_X} \imath \sigma^{\mu\nu} (q_X)_\nu P_R \right] \tag{19}$$

and the additional WXZ vertex is of the same structure as the usual WWZ vertex of the SM. This additional tXb vertex does not effect the SM's fermion cancellations of ABJ gauge anomalies.

9. Manifestation of intensity-ratio equivalence

The $t \rightarrow W^+ b$ helicity decay amplitudes in the SM and those with the additional (+) coupling of Eqs.(2, 18) are related by a set of transformation matrices. Similar to matrix representations of Lie Groups, these matrices form various algebras and sub-algebras, with associated symmetries which should be useful in investigating deeper dynamics in top quark physics [1,2,5].

This is an analytic generalization of the many numerical patterns in Table 1, i.e. it isn't merely 2 isolated amplitudes being related per Eq.(17). Instead, this makes manifest the intensity-ratio equivalence of Sec. 7. The tWb-transformations are: $A_+ = vM\ A_{SM},\ vP\ A_{SM}, \dots$; $A_{SM} = v^{-1}M\ A_+,\ -v^{-1}P\ A_+, \dots$; v is the W-boson velocity in the t-quark rest frame; with $\Lambda_+ = E_W/2$ but unfixed values of m_W/m_t and m_b/m_t. Figs. (4-5) give a compact and complete summary.

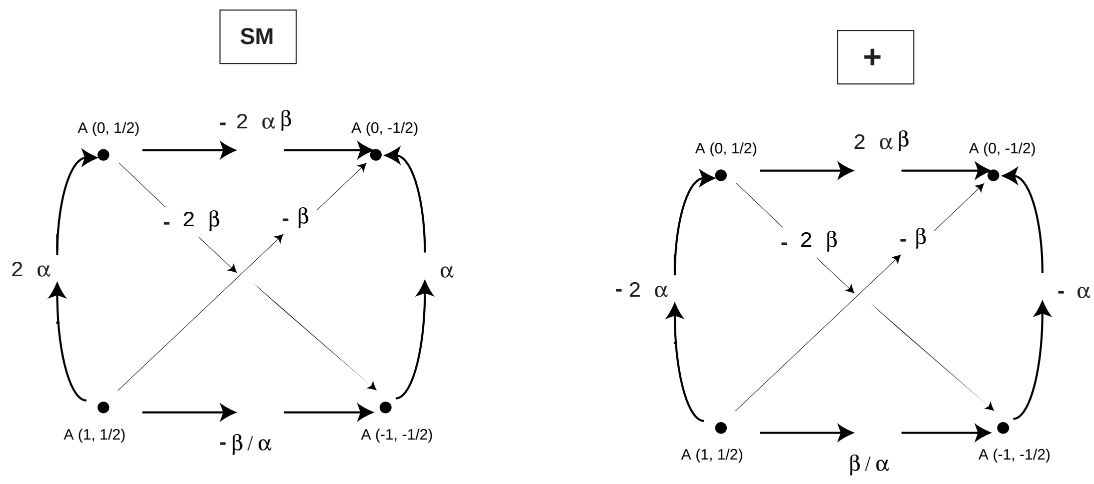

Figure 4: Left: Self-Transformation factors between the helicity amplitudes of the Standard Model. Right: Self-Transformation factors between the helicity amplitudes of the (+) model; note the sign changes in the outer four factors versus those for the SM.

With $\alpha = a/v, \beta = b/v$, these matrices are: $M = diag(1, -1, -1, 1)$,

$$P(\alpha) \equiv \begin{bmatrix} 0 & \alpha & 0 & 0 \\ -1/\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2\alpha \\ 0 & 0 & 2\alpha & 0 \end{bmatrix}, B(\beta) \equiv \begin{bmatrix} 0 & 0 & 0 & -\beta \\ 0 & 0 & 2\beta & 0 \\ 0 & 1/2\beta & 0 & 0 \\ -1/\beta & 0 & 0 & 0 \end{bmatrix}, \tag{20}$$

$$G(\alpha, \beta) \equiv \begin{bmatrix} 0 & 0 & -2\alpha\beta & 0 \\ 0 & 0 & 0 & \beta/\alpha \\ 1/2\alpha\beta & 0 & 0 & 0 \\ 0 & -\alpha/\beta & 0 & 0 \end{bmatrix}, Q(\alpha) \equiv \begin{bmatrix} 0 & \alpha & 0 & 0 \\ 1/\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2\alpha \\ 0 & 0 & 2\alpha & 0 \end{bmatrix}, \tag{21}$$

$$C(\beta) \equiv \begin{bmatrix} 0 & 0 & 0 & \beta \\ 0 & 0 & 2\beta & 0 \\ 0 & 1/2\beta & 0 & 0 \\ 1/\beta & 0 & 0 & 0 \end{bmatrix}, H(\alpha, \beta) \equiv \begin{bmatrix} 0 & 0 & 2\alpha\beta & 0 \\ 0 & 0 & 0 & \beta/\alpha \\ 1/2\alpha\beta & 0 & 0 & 0 \\ 0 & \alpha/\beta & 0 & 0 \end{bmatrix}, \tag{22}$$

Including the identity matrix, this is the closed 8-element transformation algebra with a commutator/anticommutator structure. It has $7 = 4_- + 3_+$ three-element closed subalgebras ($\mp$ subscript denotes non-trivial commutators/anticommutators). There are 2x2 matrix compositions of the above matrices with associated commutator/anticommutator algebras.

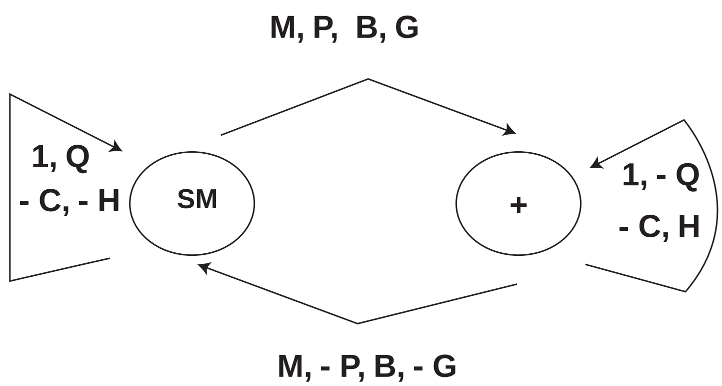

Figure 5: Diagram displaying the roles of all the matrices in transforming the helicity amplitudes of either the SM or the (+) model. The matrices in the diagram form the 8-element closed transformation algebra. Included are the sign factors needed in order to get the correct transformation result using each of the matrices.

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References

[1] Three Numerical Puzzles and the Top Quark's Chiral Weak-Moment ; C.A. Nelson, Phys. Rev. **D65**, 074033(2002); hep-ph/0106138.

[2] Consequences of a Large Top-Quark Chiral Weak-Moment ; C.A. Nelson, p. 319 in *Results and Perspectives in Particle Physics*, M. Greco (Ed) (INFN, Frascatti, Roma, Italy, 2003), p. 319; hep-ph/0411072.

[3] Use of W-boson Longitudinal-Transverse Interference in Top Quark Spin-Correlation Functions ; C.A. Nelson, E.G. Barbagiovanni, J.J. Berger, E.K. Pueschel, J.R. Wickman, Eur. Phys. J. **C45**, 121(2006); hep-ph/0506240.

[4] Use of W-boson Longitudinal-Transverse Interference in Top Quark Spin-Correlation Functions: II ; C.A. Nelson, J.J. Berger, J.R. Wickman, Eur. Phys. J. **C46**, 385(2006); hep-ph/0510348. To emphasize the common final-state interference structure in the BR-S2SC, we have defined for Eqs.(11,12): $B_0^q(s,\Theta_B) = \frac{1}{24}\widetilde{g}_1^q(s,\Theta_B)$, $B_{sig}^q(s,\Theta_B) = \frac{1}{24}\widetilde{g}_2^q(s,\Theta_B)$, $B_0^g(s,\Theta_B) = \overline{c}(s,\Theta_B)\widetilde{g}_1^g(s,\Theta_B)$, $B_{sig}^g(s,\Theta_B) = \overline{c}(s,\Theta_B)\widetilde{g}_2^g(s,\Theta_B)$, with $\overline{c}(s,\Theta_B) = 3c(s,\Theta_B)$.

[5] C.A. Nelson and J.J. Berger, (joint research); Transformation Algebras between a New Coupling in Top Quark Decay and the Standard Model, J.J. Berger, Senior Honors Thesis at SUNY at Binghamton (May 2007).